EXAMPLE SHEET

Exercise on thermo-fluid dynamics, June 13-14, 2016

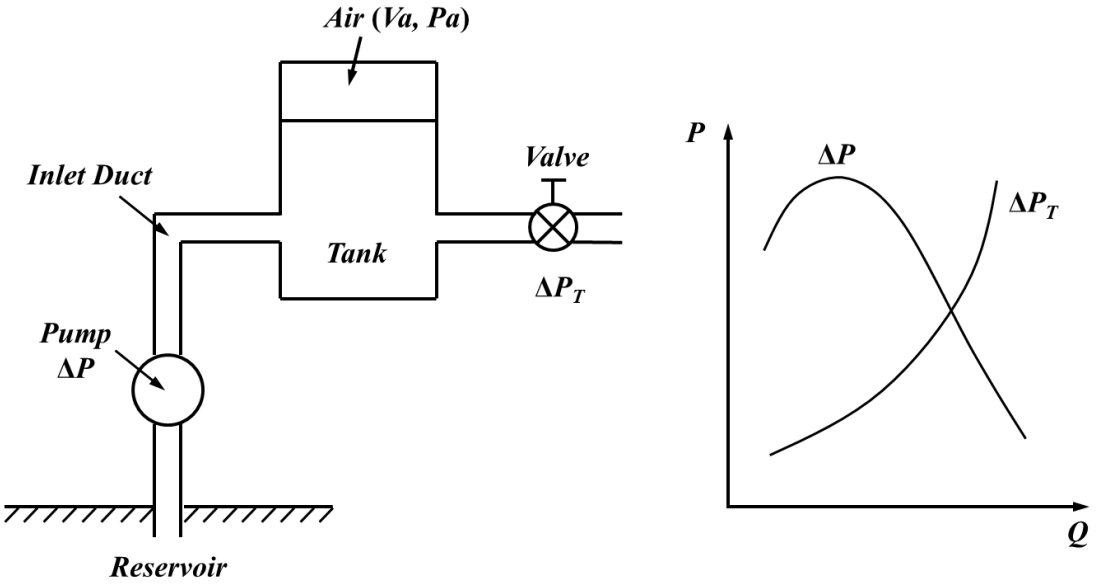
1、 Reservoir water is pumped up via an inlet duct of length L and cross section area A into a tank, whence is exhausted to outside through a throttle valve.

The characteristics of pump pressure head ΔP and exhaust valve loss ΔP_T are given with respect to volume flow rate Q.

The system is observed to operate at a steady tank condition with partially filled air of volume Va and pressure Pa.

Examine the characteristic frequency of this flow system by assuming that the air is trapped in the tank to follow isentropic change of state, and gravity effect and the difference in kinetic energy of water through the inlet duct is negligible.

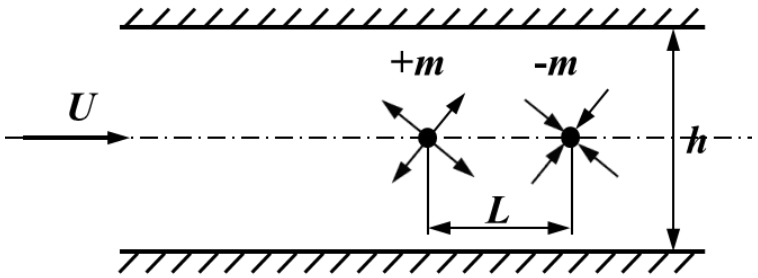
蓄水池的水通过一个进口长度为 L，横截面积为 A 的管道抽到水箱，而后通过一个节流阀排出。泵的压头 ΔP 和排水阀损失 ΔP_T 与体积流量 Q 相关。从下图可以看到，系统在运行时，稳定的水箱顶部填充了部分空气。空气体积为 Va，压力为 Pa。假设空气限制在水箱中，且满足等熵状态变化，忽略重力效应和通过进口管道后水的动能损失，分析该流动系统的频率特性。



2、 A couple of source and sink of strength $\pm m$ is placed with a short distance L at the centre line of wind tunnel between two parallel walls of separation h.

Supposing the extreme of L to become infinitesimal, keeping the multiplier $\mu=mL$ to be constant, what is the equivalent shape of a stream body represented by that singularity point in the presence of uniform wind speed U? Examine carefully the effect of interference with the wind tunnel walls.

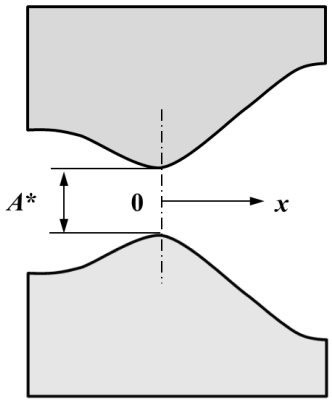
两个强度分别为 $\pm m$ 的源和汇以一个较小的距离 L 放置在由两个距离为 h 的平行壁面构成的风洞中心线上。假设 L 的极限为无穷小，保持 $\mu=mL$ 为常数。那么，这种奇点在均匀风速 U 中所代表的流线性物体的等效形状是什么？仔细分析风洞壁面的干涉效应。



3、 A Laval nozzle (convergent and divergent) is operating at critical choked condition. Geometry near the throat is approximately given by parabolic curve along the nozzle axis x, i.e. the cross section area $A=A^*(1+x^2)$, *indicating the throat position.

Find the rate of Mach number increase $\mathrm{d}M/\mathrm{d}x$ at the throat.

一个拉法尔喷管（缩放喷管）在临界堵塞条件下运行。靠近喉部的几何由沿喷管轴线 x 的抛物线分布规律近似给定，例如，横截面积 $A=A^*(1+x^2)$，*代表喉部位置。求在喉部处的马赫数增加速率 $\mathrm{d}M/\mathrm{d}x$。



4、 Suppose that heat is locally added to uniform inviscid flow in a duct, where the state of static pressure P_1, density ρ_1, velocity u_1, Mach number M_1 and cross section area A_1 are all given at the entrance of the heating zone, and the fluid specific heats C_P, C_V and their ratio $\gamma = C_P/C_V$ are assumed constant (perfect gas).

a) Gas turbine combustor: Find the area A_2 and Mach number M_2 at the exit of the heating zone in case the pressure is kept constant through the heat process. What is the entropy increase $\Delta s = s_2 - s_1$ and total pressure ratio $\pi = P_{t2}/P_{t1}$ across the heating zone in terms of heating rate q (amount of heat per mass flow rate)?

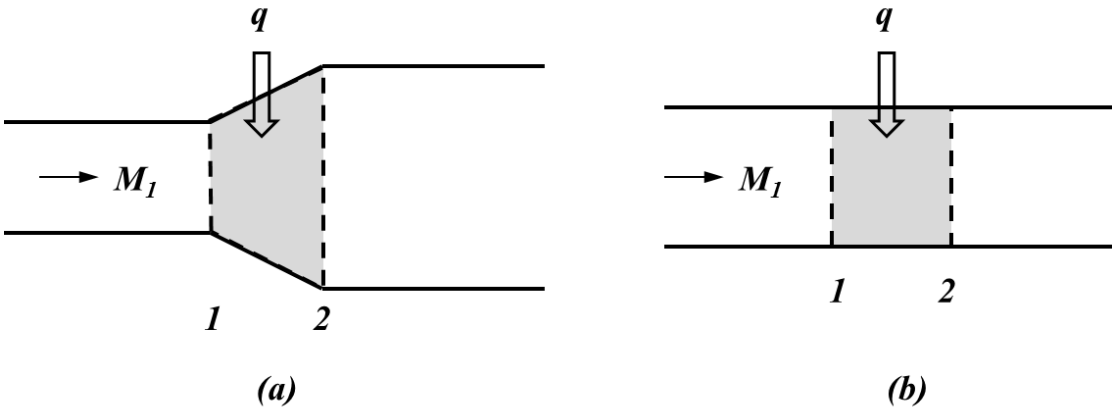
b) Rayleigh flow: If the heating zone area is kept constant, find the heating rate q

to achieve sonic condition at the zone exit?

假设热量局部添加到一个管道的均匀无粘流体中，在加热区的进口给定流体静压 P_1，密度 ρ_1，速度 u_1，马赫数 M_1 和横截面积 A_1，假设流体比热 C_P，C_V 和比热比 $\gamma = C_P/C_V$ 为常数（完全气体）。

a) 燃气轮机燃烧室：假设加热过程中压力保持常数，求加热区出口处的横截面积 A_2 和马赫数 M_2。通过加热区的熵增 $\Delta s = s_2 - s_1$ 和总压比 $\pi = P_{t2}/P_{t1}$ 用加热率 q（单位质量流量的加热量）表示是多少？

b) 瑞利流动(Rayleigh flow)：如果加热区面积保持为常数，求在出口达到音速条件时的加热率 q?



5、 A centrifugal compressor is operating with impeller tip speed u_t (tip diameter $=D_t$ and width$=b_t$). A diffuser passage of width $b(r)$ is attached downstream. In the rotor coordinates, gas velocity w_t at the tip has its components $+w_{rt}$ and $-w_{\theta t}$ in the radial and circumferential (positive towards rotor motion) direction, respectively.

a) Given the static enthalpy h_t of the gas at the impeller tip, find the radial velocity $w_r(r)$, circumferential velocity $w_\theta(r)$ as well as static enthalpy $h(r)$ of the gas at any radial position r. Assume the flow is isentropic and its speed is low in comparison with sonic speed.

b) Find corresponding $w_r(r)$, $w_\theta(r)$ and static enthalpy $h(r)$, when the tip velocity w_t becomes comparable with sonic speed.

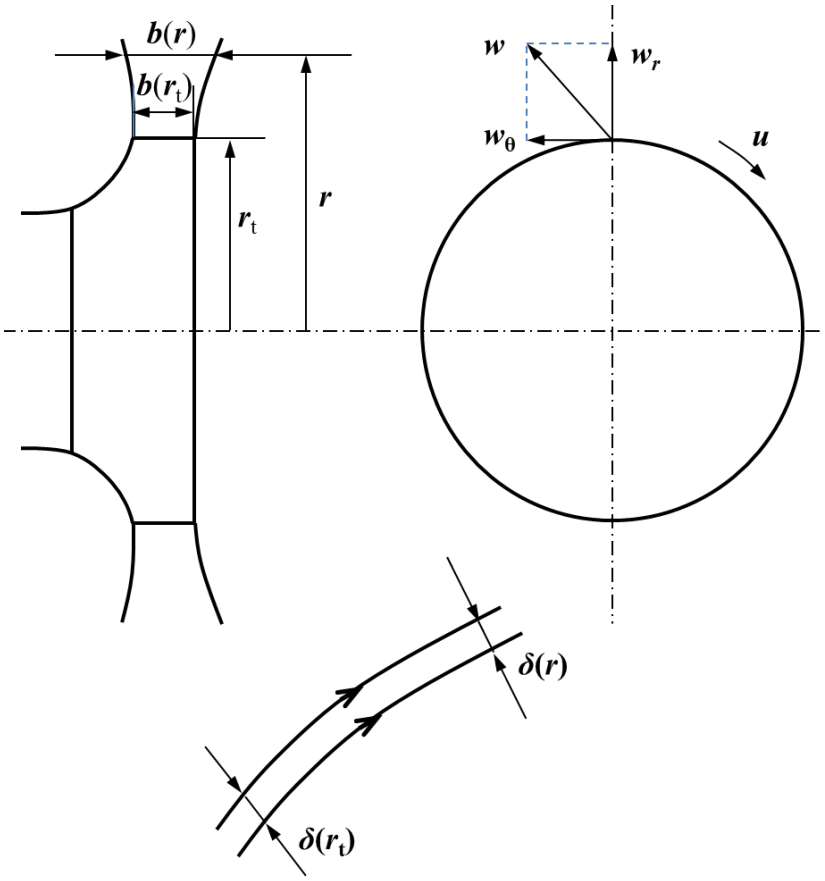
c) Examine change in the distance between neighboring streamlines i.e. the ratio $\delta(r)/\delta t$, with respect to either relative or absolute coordinates.

离心压气机以圆周速度 u_t 运行（出口直径 D_t，宽度 b_t）。下游连接一个宽度为 $b(r)$的扩压器通道。在叶轮旋转坐标系下，叶轮出口气体速度 w_t 分别具有径向分量$+w_{rt}$ 和周向分量$-w_{\theta t}$（沿旋转方向速度分量为正）。

a) 给定叶轮出口处气体的静焓 h_t，求在任意径向位置 r 处气体的径向速度 $w_r(r)$，周向速度 $w_\theta(r)$和静焓 $h(r)$。假设流动是等熵的，且速度低于声速。

b) 当出口速度 w_t 接近音速时，求相应的 $w_r(r)$, $w_\theta(r)$ 和 $h(r)$。

c) 分析相邻流线之间距离的变化，例如，相对坐标系或绝对坐标系下的 $\delta(r)/\delta t$。


b(r)
b(rt)
r
rt
w
wr
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u
δ(r)
δ(rt)